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Entanglement Conditions for Mixed SU(2) and SU(1, 1) Systems

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Abstract We derive a class of inequalities for detecting entanglement in the mixed SU(2) and SU(1, 1) systems based on the Schrödinger-Robertson indeterminacy relations in conjugation with the partial transposition. These inequalities are in general stronger than those based on the usual Heisenberg uncertainty relations for detecting entanglement. Furthermore, based on the complete reduction from SU(2) and SU(1, 1) systems to bosonic systems, we derive some entanglement conditions for two-mode systems. We also use the partial reduction to obtain some inequalities in the mixed SU(2) (or SU(1, 1)) and bosonic systems.

Keywords Entanglement conditions · Schrödinger-Robertson indeterminacy relations · Heisenberg uncertainty relations · Partial transposition · Reduction

1 Introduction

Entanglement is a key element for quantum information processing. Consequently, an important problem in quantum information theory [1, 2] is the formulation of appropriate methods for detecting entanglement and then finding measures that quantify the degree of entanglement. However, determining whether or not a quantum state is entangled is often far

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from simple. Some methods of entanglement conditions exist indeed, for instance, Peres-Horodecki positive partial transpose (PPT) condition [3, 4], entanglement witnesses [5], and hierarchies of entanglement conditions [6], but they are not always direct to apply.

In particular, for infinite dimensional systems, i.e., the continuous variable (CV) systems, the conditions of entanglement are very limited. In recent years, Gaussian state entanglement, a special class of state of CV systems, has aroused great interest [7–12]. There has been some progress in the study of entanglement conditions for non-Gaussian states by Heisenberg uncertainty relations (HUR) [13–18]. Hillery et al. have provided a class of inequalities for detecting entanglement of two-mode states [16]. Furthermore Nha et al. derived a class of inequalities, which are able to detect non-Gaussian entangled states, from the HUR of the SU(1, 1) and the SU(2) algebra in conjunction with partial transposition (PT) [19]. Soon, Nha et al. derived a class of stronger bound inequalities than the previous context via the Schrödinger-Robertson indeterminacy relation (SRIR) and PT [20].

For observables A and B, which obey the commutation relations $[A, B] = iC$, we have the inequalities

$$\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq \frac{1}{4}|\langle C\rangle|^2 \tag{1}$$

called HUR. This uncertainty relation contains essential physics from the points of view of experiments, and it is a very fundamental element in quantum mechanics. The term of $\langle(\Delta A)^2\rangle$ denotes the variance or the uncertainty of the observable A and inequality represents that the production of the two variances which are noncommuting, in principle, has a lower bound.

On the other hand, a stronger bound is given by

$$\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq \frac{1}{4}|\langle C\rangle|^2 + \langle\Delta A\Delta B\rangle^2, \tag{2}$$

where the cross correlation $\langle\Delta A\Delta B\rangle$ is defined in a symmetric form as

$$\langle\Delta A\Delta B\rangle = \frac{1}{2}\langle\Delta A\Delta B + \Delta B\Delta A\rangle. \tag{3}$$

It is easy to check

$$\langle\Delta A\Delta B\rangle = \frac{1}{2}\langle AB + BA\rangle - \langle A\rangle\langle B\rangle = \mathrm{Cov}(A, B), \tag{4}$$

namely, the cross correlation is nothing but the covariance.

In this paper, we derive, firstly, a class of inequalities for the mixed systems via using the HUR and SRIR with PT. The system is a mixed one between SU(2) and SU(1, 1) subsystems. We also consider the reduction from SU(2) (SU(1, 1)) to bosonic systems. This will leads to new entanglement inequalities for mixed systems such as spin-boson, SU(1, 1)-boson systems as well as the two-mode bosonic systems.

This paper is organized as follows. In Sect. 2 we construct the operators corresponding to the mixed system and then derive a class of inequalities via the HURs and SRIRs with PT. In Sect. 3 we present not only the complete reductions of inequalities in these mixed system for detecting entanglement, but also the entanglement criteria by partial reductions. Finally, we summarize our results in Sect. 4.

2 Entanglement Conditions for Mixed Systems

In this section, we apply the PT and the uncertainty relations to the following mixed system to obtain entanglement inequalities.

2.1 Entanglement Conditions by HUR

The system we are considering is a mixed one, and one subsystem is described by the su(2) algebra, and another by su(1, 1) algebra. First, we define the observable operators of the mixed system

$$\begin{aligned} A_x &= \frac{1}{2}(K_+J_- + K_-J_+), \\ A_y &= \frac{1}{2i}(K_+J_- - K_-J_+), \\ A_z &= \frac{1}{2}(K_+K_-J_-J_+ - K_-K_+J_+J_-). \end{aligned} \tag{5}$$

Here, $J_\pm = J_x \pm iJ_y$, satisfying $[J_+, J_-] = 2J_z$. $J_\pm$ and J_z are generators of su(2) algebra. The su(1, 1) algebra is generated by the operators K_x, K_y, and K_z which satisfy the commutation relations $[K_x, K_y] = -iK_z$, $[K_+, K_-] = -2K_z$. It is straightforward to test the following commutation relations $[A_x, A_y] = iA_z$ and $A_\alpha = A_\alpha^\dagger(\alpha = x, y, z)$. As we all know, there are some realizations of su(1, 1) and su(2) Lie algebras. For example, there are two-mode realization which is realized in optics using two-mode fields, four-mode realization, Holstein-Primakoff realization of su(1, 1) and so on. On the other hand, the su(2) Lie algebra can deal with angular momentum operators. Hence, the operators A_α can describe a class of physical mixed systems.

Now, we apply the theory of the partial transposition to the mixed system above. Considering any operators A, we know that its expectation value on the partially transposed density operator satisfies

$$\langle A\rangle_{\rho^{\text{pt}}} = \langle A^{\text{pt}}\rangle_\rho, \tag{6}$$

where ρ and ρ^{pt} denote density operator and its partial transposition, respectively. We need another set of operators $\tilde{A}_x$ and $\tilde{A}_y$ can be obtained from applying the partial transposition to A_x and A_y, i.e., $\tilde{A}_x = A_x^{\text{T}_2}$ and $\tilde{A}_y = A_y^{\text{T}_2}$, and the superscript T_2 denotes the partial transposition to SU(2) system. The operators are given by

$$\begin{aligned} \tilde{A}_x &= \frac{1}{2}(K_+J_+ + K_-J_-), \\ \tilde{A}_y &= \frac{1}{2i}(K_+J_+ - K_-J_-), \\ \tilde{A}_z &= \frac{1}{2}(K_+K_-J_+J_- - K_-K_+J_-J_+), \end{aligned} \tag{7}$$

satisfying $[\tilde{A}_x, \tilde{A}_y] = i\tilde{A}_z$.

Then we have $\langle(\Delta\tilde{A}_x)^2\rangle_\rho\langle(\Delta\tilde{A}_y)^2\rangle_\rho \geq \frac{1}{4}|\langle\tilde{A}_z\rangle_\rho|^2$ with respect to the operators $\tilde{A}_\alpha$ ($\alpha = x, y, z$), which must be satisfied by any quantum states, from the commutation $[\tilde{A}_x, \tilde{A}_y] = i\tilde{A}_z$ by considering HUR. If we choose a separable state, which is represented by a density operators ρ^{sep}, it remains physical after partial transposition [4]. Consequently, we have the following inequality

$$(\Delta\tilde{A}_x)^2_{\rho^{\mathrm{T}_2}}(\Delta\tilde{A}_y)^2_{\rho^{\mathrm{T}_2}} \geqslant \frac{1}{4}|\langle\tilde{A}_z\rangle_{\rho^{\mathrm{T}_2}}|^2. \tag{8}$$

This inequality is valid for any separable state, thus the violation of it means the existence of quantum entanglement.

Using (6), we find that $(\Delta A_x)^2_{\rho^{T_2}}$ is related to the $(\Delta\tilde{A}_x)^2_\rho$ as

$$\begin{aligned}(\Delta\tilde{A}_x)^2_{\rho^{\mathrm{T}_2}} &\equiv \langle\tilde{A}_x^2\rangle_{\rho^{\mathrm{T}_2}} - \langle\tilde{A}_x\rangle^2_{\rho^{\mathrm{T}_2}} \\ &= \langle(\tilde{A}_x^2)^{\mathrm{T}_2}\rangle_\rho - \langle\tilde{A}_x^{\mathrm{T}_2}\rangle^2_\rho \\ &= \langle A_x^2 - K_zJ_z\rangle_\rho - \langle A_x\rangle^2_\rho \\ &= (\Delta A_x)^2_\rho - \langle K_zJ_z\rangle_\rho.\end{aligned} \tag{9}$$

Similarly, we have

$$(\Delta\tilde{A}_y)^2_{\rho^{\mathrm{T}_2}} = (\Delta A_y)^2_\rho - \langle K_zJ_z\rangle_\rho. \tag{10}$$

By considering simultaneously (8), (9) and (10), we obtain the inequality below

$$[(\Delta A_x)^2_\rho - \langle K_zJ_z\rangle_\rho] \times [(\Delta A_y)^2_\rho - \langle K_zJ_z\rangle_\rho] \geqslant \frac{1}{4}|\langle\tilde{A}_z\rangle_\rho|^2, \tag{11}$$

where we have used $\langle\tilde{A}_z\rangle_{\rho^{\mathrm{T}_2}} = \langle\tilde{A}_z\rangle_\rho$. So, a state is entangled if it satisfies $[(\Delta A_x)^2_\rho - \langle K_zJ_z\rangle_\rho] \times [(\Delta A_y)^2_\rho - \langle K_zJ_z\rangle_\rho] - \frac{1}{4}|\langle A_z\rangle_\rho|^2 < 0$.

Also, by using the inequality $x^2 + y^2 \geqslant 2xy$ and (9), (10), another inequality is obtained as

$$(\Delta A_x)^2_\rho + (\Delta A_y)^2_\rho \geqslant |\langle\tilde{A}_z\rangle_\rho| + 2\langle K_zJ_z\rangle_\rho. \tag{12}$$

This equation provides another entanglement criteria.

2.2 Entanglement Conditions by SRIR

Here, we consider SRIR other than HUR in conjugation with PT, and another set of inequalities are given by

$$[(\Delta A_x)^2_\rho - \langle K_zJ_z\rangle_\rho] \times [(\Delta A_y)^2_\rho - \langle K_zJ_z\rangle_\rho] \geqslant \frac{1}{4}|\langle\tilde{A}_z\rangle_\rho|^2 + \langle\Delta A_x\Delta A_y\rangle^2_\rho, \tag{13}$$

$$(\Delta A_x)^2_\rho + (\Delta A_y)^2_\rho \geqslant 2\left[\left(\frac{1}{4}|\langle\tilde{A}_z\rangle_\rho|^2 + \langle\Delta A_x\Delta A_y\rangle^2_\rho\right)^{\frac{1}{2}} + \langle K_zJ_z\rangle_\rho\right], \tag{14}$$

in principle, because the term $\langle\Delta A_x\Delta A_y\rangle^2_\rho$ is nonnegative, equations (13) and (14) are stronger than equations (11) and (12), respectively. Specifically, $\langle\Delta A_x\Delta A_y\rangle^2_\rho = 0$, which means that the covariance of A and B are zeros. In this case, the SRIRs reduce to the HURs.

Using the same method and starting from the commutation relation $[A_x, A_y] = iA_z$, we obtain another set of inequalities which are represented by operators $\tilde{A}_\alpha$ $(\alpha = x, y, z)$

$$[(\Delta\tilde{A}_x)^2_\rho + \langle K_zJ_z\rangle_\rho] \times [(\Delta\tilde{A}_y)^2_\rho + \langle K_zJ_z\rangle_\rho] \geqslant \frac{1}{4}|\langle A_z\rangle_\rho|^2 + \langle\Delta\tilde{A}_x\Delta\tilde{A}_y\rangle^2_\rho, \tag{15}$$

$$(\Delta\tilde{A}_x)^2_\rho + (\Delta\tilde{A}_y)^2_\rho \geqslant 2\left[\left(\frac{1}{4}|\langle A_z\rangle_\rho|^2 + \langle\Delta\tilde{A}_x\Delta\tilde{A}_y\rangle^2_\rho\right)^{\frac{1}{2}} - \langle K_z J_z\rangle_\rho\right]. \tag{16}$$

3 Reduction Using Holstein-Primakoff Transformation

It is well known that the operators

$$J_+ = a^\dagger\sqrt{2j - N_a}, \qquad J_- = \sqrt{2j - N_a}a, \qquad J_z = N_a - j, \tag{17}$$

generate the su(2) algebra via the Holstein-Primakoff (H-P) representation [21–23] in the spin-j representation. We have

$$\frac{J_+}{\sqrt{2j}} \to a^\dagger, \qquad \frac{J_-}{\sqrt{2j}} \to a, \qquad \frac{J_z}{j} \to -1 \tag{18}$$

in the large j limit and small excitations. Bosonic mode is represented by the annihilation operators a or b. Similarly, the generators of the su(1, 1) algebra, obtained via the Holstein-Primakoff realization of the discrete irreducible representation with Bargmann index k, are

$$K_+ = a^\dagger\sqrt{2k + N_a}, \qquad K_- = \sqrt{2k + N_a}a, \qquad K_z = N_a + k$$

with

$$\frac{K_+}{\sqrt{2k}} \to a^\dagger, \qquad \frac{K_-}{\sqrt{2k}} \to a, \qquad \frac{K_z}{k} \to 1.$$

In the large k limit and small excitations. Here, $N_a = a^\dagger a$ is the number operator. The reduction can be obtained by expanding the square root and neglecting terms of $O(1/j)$ ($O(1/k)$). We see that the bosonic system and SU(2) (SU(1, 1)) systems are connected by the large j (k) limit from an algebraic point of view.

3.1 Complete Reduction from the Mixed Systems to Bosonic Systems

We apply H-P transformation to the inequalities which are obtained from the previous context. For instance, we choose (13). Starting from

$$(\Delta A_\alpha)^2_\rho/4jk \to (\Delta\mathcal{J}_\alpha)^2_\rho \quad (\alpha = x, y), \tag{19}$$

$$\langle K_z J_z\rangle_\rho/4jk \to -\frac{1}{4}, \tag{20}$$

$$\frac{1}{4}|\langle\tilde{A}_z\rangle_\rho|^2/16j^2k^2 \to \frac{1}{16}|1 + \langle N_a + N_b\rangle|^2, \tag{21}$$

$$\langle\Delta A_x\Delta A_y\rangle^2_\rho/16j^2k^2 \to \langle\Delta\mathcal{J}_x\Delta\mathcal{J}_y\rangle^2_\rho \tag{22}$$

and letting $j, k \to \infty$, we have

$$[4(\Delta\mathcal{J}_x)^2_\rho + 1] \times [4(\Delta\mathcal{J}_x)^2_\rho + 1] \geqslant |1 + \langle N_a + N_b\rangle|^2 + 16\langle\Delta\mathcal{J}_x\Delta\mathcal{J}_y\rangle^2_\rho. \tag{23}$$

Here,

$$\mathcal{J}_x = \frac{1}{2}(a^\dagger b + ab^\dagger),$$
$$\mathcal{J}_y = \frac{1}{2i}(a^\dagger b - ab^\dagger), \tag{24}$$
$$\mathcal{J}_z = \frac{1}{2}N_-$$

are two-mode realization of the su(2) algebra. We find that inequality (13) for mixed SU(2) and SU(1, 1) system reduces to inequality (23) for the bosonic system in the limit of j, $k \to \infty$ and it is the same as the inequality obtained in [20]. Thus, one can use (23), which are obtained by complete reduction, to detect entanglement in the bosonic system.

If we choose (15), after reduction, we obtain another entanglement criteria for two-mode systems

$$[4(\Delta\mathcal{K}_x)^2_\rho - 1] \times [4(\Delta\mathcal{K}_x)^2_\rho - 1] \geqslant |\langle N_a - N_b\rangle|^2 + 16\langle\Delta\mathcal{K}_x\Delta\mathcal{K}_y\rangle^2_\rho, \tag{25}$$

where

$$\mathcal{K}_x = \frac{1}{2}(a^\dagger b^\dagger + ab),$$
$$\mathcal{K}_y = \frac{1}{2i}(a^\dagger b^\dagger - ab), \tag{26}$$
$$\mathcal{K}_z = \frac{1}{2}(N_+ + 1)$$

are two-mode realization of su(1, 1) algebra and $N_+ = N_a + N_b$. This is another entanglement criteria for two-mode systems.

3.2 Partial Reduction from Mixed SU(2) and SU(1, 1) Systems to Mixed SU(2) (or SU(1, 1)) and Bosonic Systems

Here, we continue using the usual H-P transformation to study inequalities for detecting entanglement. For a mixed SU(2) and SU(1, 1) system, if we don't reduce it completely, the mixed system will become another mixed system which are mixed SU(2) (or SU(1, 1)) and bosonic subsystem. Then after reduction, we can obtain entanglement conditions for these systems.

3.2.1 Reduction from Mixed SU(2) and SU(1, 1) System to Mixed SU(1, 1) and Bosonic Systems

Here, we keep K_α ($\alpha = x, y, z$) and reduce J_α ($\alpha = x, y, z$) in the limits of $j \to \infty$. So we have

$$\frac{\tilde{A}_x}{\sqrt{2j}} \to B_x = \frac{1}{2}(K_+ a^\dagger + K_- a),$$
$$\frac{\tilde{A}_y}{\sqrt{2j}} \to B_y = \frac{1}{2i}(K_+ a^\dagger - K_- a), \tag{27}$$
$$\frac{\tilde{A}_z}{2j} \to B_z = \frac{1}{2}(K_+ K_- a^\dagger a - K_- K_+ a a^\dagger),$$

where $[B_x, B_y] = iB_z$. According to the previous discussion,we must build another set of operators by partial transposition, and they are given by

$$\begin{aligned}\tilde{B}_x &= \frac{1}{2}(K_+a + K_-a^\dagger),\\ \tilde{B}_y &= \frac{1}{2i}(K_+a - K_-a^\dagger),\\ \tilde{B}_z &= \frac{1}{2}(K_+K_-aa^\dagger - K_-K_+a^\dagger a).\end{aligned} \tag{28}$$

Consequently, from (15) and (16), in the limit of $j \to \infty$, we can have the entanglement criteria for the mixed SU(1, 1) and bosonic system

$$[2(\Delta B_x)^2_\rho - \langle K_z\rangle_\rho] \times [2(\Delta B_y)^2_\rho - \langle K_z\rangle_\rho] \geqslant |\langle \tilde{B}_z\rangle_\rho|^2 + 4\langle \Delta B_x \Delta B_y\rangle^2_\rho, \tag{29}$$

$$(\Delta B_x)^2_\rho + (\Delta B_y)^2_\rho \geqslant 2\left[\left(\frac{1}{4}|\langle \tilde{B}_z\rangle_\rho|^2 + \langle \Delta B_x \Delta B_y\rangle^2_\rho\right)^{\frac{1}{2}} + \frac{1}{2}\langle K_z\rangle_\rho\right], \tag{30}$$

respectively. Here, we have used the commutation relation $[a, a^\dagger] = 1$.

Also, we may continue the partial reduction from SU(1, 1) to another bosonic system. In the limit of $k \to \infty$, we have

$$\frac{B_x}{\sqrt{2k}} \to \mathcal{K}_x, \qquad \frac{B_y}{\sqrt{2k}} \to \mathcal{K}_y, \qquad \frac{B_z}{2k} \to \mathcal{K}_z, \tag{31}$$

where the operators $\mathcal{K}_\alpha$ $(\alpha = x, y, z)$ are generators of SU(1, 1) algebra. Here, from (29) and (30), we can obtain again the entanglement criteria which are given by (23) and (25), respectively.

3.2.2 Reduction from Mixed SU(2) and SU(1, 1) System to Mixed SU(2) and Bosonic Systems

Next, we make reduction of SU(1, 1) system, and in the limits of $k \to \infty$, we have

$$\begin{aligned}\frac{\tilde{A}_x}{\sqrt{2k}} &\to C_x = \frac{1}{2}(J_+a^\dagger + J_-a),\\ \frac{\tilde{A}_y}{\sqrt{2k}} &\to C_y = \frac{1}{2i}(J_+a^\dagger - J_-a),\\ \frac{\tilde{A}_z}{2k} &\to C_z = \frac{1}{2}(J_+J_-a^\dagger a - J_-J_+aa^\dagger),\end{aligned} \tag{32}$$

where $[C_x, C_y] = iC_z$. Then we build another set of operators

$$\begin{aligned}\tilde{C}_x &= \frac{1}{2}(J_+a + J_-a^\dagger),\\ \tilde{C}_y &= \frac{1}{2i}(J_+a - J_-a^\dagger),\\ \tilde{C}_z &= \frac{1}{2}(J_+J_-aa^\dagger - J_-J_+a^\dagger a),\end{aligned} \tag{33}$$

and they satisfy the commutation relation $[\tilde{C}_x, \tilde{C}_y] = i\tilde{C}_z$. Thus, the original system is changed into mixed SU(2) and bosonic system. The corresponding inequalities are obtained as

$$[2(\Delta C_x)^2_\rho + \langle J_z\rangle_\rho] \times [2(\Delta C_y)^2_\rho + \langle J_z\rangle_\rho] \geqslant |\langle \tilde{C}_z\rangle_\rho|^2 + 4\langle \Delta C_x \Delta C_y\rangle^2_\rho, \tag{34}$$

$$(\Delta C_x)^2_\rho + (\Delta C_y)^2_\rho \geqslant 2\left[\left(\frac{1}{4}|\langle \tilde{C}_z\rangle_\rho|^2 + \langle \Delta C_x \Delta C_y\rangle^2_\rho\right)^{\frac{1}{2}} - \frac{1}{2}\langle J_z\rangle_\rho\right]. \tag{35}$$

Here, we have used the commutation relation $[a, a^\dagger] = 1$.

Also, we may continue the reduction, and in the limit of $j \to \infty$, we obtain

$$\frac{\tilde{C}_x}{\sqrt{2j}} \to \mathcal{J}_x, \qquad \frac{\tilde{C}_y}{\sqrt{2j}} \to \mathcal{J}_y, \qquad \frac{\tilde{C}_z}{2j} \to \mathcal{J}_z,$$

where the operators $\mathcal{J}_\alpha (\alpha = x, y, z)$ are generators of SU(2) algebra. Similarly, from (34) and (35), we may obtain again the previous entanglement criteria which are given by (23) and (25) for two-mode systems, respectively.

4 Conclusions

In this paper, based on the uncertainty relations in conjugation with the partial transposition, we have derived a class of entanglement criteria which are able to detect entanglement in mixed SU(2) and SU(1, 1) systems. In principle, the entanglement conditions based on the SRIRs are stronger than those on HURs. No matter pure or mixed states, the entanglement condition obtained here can be applicable.

Furthermore, we have shown another way to get entanglement criteria of mixed SU(2) (or SU(1, 1)) and bosonic system by H-P transformation. We have recovered the entanglement criteria of the two-mode bosonic system which is reduced completely from the mixed SU(2) and SU(1, 1) system. By partial reductions, the inequalities are obtained for detecting entanglement of mixed SU(2) (or SU(1, 1)) and bosonic system. Our results indicate that the indeterminacy relations are very useful in obtaining entanglement conditions in mixed systems in conjugation with the partial transposition and the partial reduction.

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